Math 10A with Professor Stankova
Worksheet, Discussion #14; Wednesday, 9/27/2017
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True/False Questions

1. **TRUE** False You can tell what the domain and range of an inverse function is only from the domain and range of the original function
2. **TRUE** False The horizontal line test tells us whether a function is injective or surjective.
3. True **FALSE** The range of e^x is $[0, \infty)$.
4. True **FALSE** If $\lim_{x\to 0} f(x)$ exists, then $\lim_{x\to 0} f(x) = f(0)$.
5. **TRUE** False Extrema of a function must occur when the derivative is 0, when it doesn't exist, or at the endpoints.
6. True **FALSE** The expression 0^∞ is an indeterminate.
7. **TRUE** False The continuity law for subtraction follows from the limit law for subtraction.
8. **TRUE** False The continuity of a constant function follows from limit laws.
9. True **FALSE** The continuity law for rational functions follows only from the limit laws for ratios.
10. **TRUE** False If f is not continuous at $x = c$, then f is not differentiable at $x = c$.
11. True **FALSE** We can use the power rule to find the derivative of x^x.

Proofs

12. Prove that if a function is differentiable at a point, then it is continuous there.

Solution: Suppose that f is differentiable at $x = c$, then we have that

$$\lim_{x\to c} \frac{f(x) - f(c)}{x - c} = f'(c).$$

Therefore, by the product limit law, we have that

$$\lim_{x\to c}[f(x) - f(c)] = \lim_{x\to c} \frac{f(x) - f(c)}{x - c} \cdot \lim_{x\to c}(x - c) = f'(c) \cdot 0 = 0.$$

By the subtraction limit law, we have that

$$0 = \lim_{x \to c}[f(x) - f(c)] = \lim_{x \to c} f(x) - \lim_{x \to c} f(c) = \lim_{x \to c} f(x) - f(c).$$

Therefore $\lim_{x \to c} f(x) = f(c)$ and so f is continuous at $x = c$.

13. Prove that if f, g are continuous at $x = c$, then $f(x)g(x)$ is continuous at $x = c$.

Solution: We can use the product limit law to get

$$\lim_{x \to c} f(x)g(x) = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x) = f(c) \cdot g(c).$$

14. Prove the addition continuity law.

Solution: We use the addition limit law to get that

$$\lim_{x \to c}[f(x) + g(x)] = \lim_{x \to c} f(x) + \lim_{x \to c} g(x) = f(c) + g(c).$$

The last inequality comes from the fact that f, g are continuous at c.